SUMMER PREPARATORY WORK

AS LEVEL MATHS

Central
Saint Michael's
Sixth Form

A UNIVERSITY-STYLE SIXTH FORM

Instructions to all students:

1. Please do the examples 1 to 14 (There are some challenges along the way)
2. Then do the Mixed Exercise questions 1 to 24 (which you will find towards the end of the booklet)

1.1 Index laws

- **You can use the laws of indices to simplify powers of the same base.**
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^n b^n$

Example 1

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $\frac{b^7}{b^4}$ **d** $6x^5 \div 3x^3$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

Example 2

Expand these expressions and simplify if possible:

a $-3x(7x - 4)$ **b** $y^2(3 - 2y^3)$

c $4x(3x - 2x^2 + 5x^3)$ **d** $2x(5x + 3) - 5(2x + 3)$

Example 3

Simplify these expressions:

a $\frac{x^7 + x^4}{x^3}$ **b** $\frac{3x^2 - 6x^5}{2x}$ **c** $\frac{20x^7 + 15x^3}{5x^2}$

1.2 Expanding brackets

To find the **product** of two expressions you **multiply** each term in one expression by each term in the other expression.

Multiplying each of the 2 terms in the first expression by each of the 3 terms in the second expression gives $2 \times 3 = 6$ terms.

$x \times$

$$\begin{aligned}(x+5)(4x-2y+3) &= x(4x-2y+3) + 5(4x-2y+3)\\ &= 4x^2 - 2xy + 3x + 20x - 10y + 15\\ &= 4x^2 - 2xy + 23x - 10y + 15\end{aligned}$$

$5 \times$

Simplify your answer by collecting like terms.

Example 4

Expand these expressions and simplify if possible:

a $(x + 5)(x + 2)$ **b** $(x - 2y)(x^2 + 1)$ **c** $(x - y)^2$ **d** $(x + y)(3x - 2y - 4)$

Example 5

Expand these expressions and simplify if possible:

a $x(2x + 3)(x - 7)$ **b** $x(5x - 3y)(2x - y + 4)$ **c** $(x - 4)(x + 3)(x + 1)$

Challenge

Expand and simplify $(x + y)^4$.

1.3 Factorising

You can write expressions as a **product of their factors**.

- **Factorising is the opposite of expanding brackets.**

Expanding brackets

$4x(2x + y) = 8x^2 + 4xy$

$(x + 5)^3 = x^3 + 15x^2 + 75x + 125$

$(x + 2y)(x - 5y) = x^2 - 3xy - 10y^2$

Factorising

Example 6

Factorise these expressions completely:

a $3x + 9$ **b** $x^2 - 5x$ **c** $8x^2 + 20x$ **d** $9x^2y + 15xy^2$ **e** $3x^2 - 9xy$

- **A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.**

To factorise a quadratic expression:

- Find two factors of ac that add up to b — For the expression $2x^2 + 5x - 3$, $ac = -6 = -1 \times 6$ and $-1 + 6 = 5 = b$.
- Rewrite the b term as a sum of these two factors — $2x^2 - x + 6x - 3$
- Factorise each pair of terms — $= x(2x - 1) + 3(2x - 1)$
- Take out the common factor — $= (x + 3)(2x - 1)$

- $\boldsymbol{x^2 - y^2 = (x + y)(x - y)}$

Example 7

Factorise:

a $x^2 - 5x - 6$ **b** $x^2 + 6x + 8$ **c** $6x^2 - 11x - 10$ **d** $x^2 - 25$ **e** $4x^2 - 9y^2$

Example 8

Factorise completely:

a $x^3 - 2x^2$ **b** $x^3 - 25x$ **c** $x^3 + 3x^2 - 10x$

Challenge

Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

1.4 Negative and fractional indices

Indices can be negative numbers or fractions.

$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^1 = x,$

similarly $\underbrace{x^{\frac{1}{n}} \times x^{\frac{1}{n}} \times \ldots \times x^{\frac{1}{n}}}_{n \text{ terms}} = x^{\frac{1}{n}+\frac{1}{n}+\ldots+\frac{1}{n}} = x^1 = x$

- **You can use the laws of indices with any rational power.**
  - $a^{\frac{1}{m}} = \sqrt[m]{a}$
  - $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
  - $a^{-m} = \dfrac{1}{a^m}$
  - $a^0 = 1$

Example 9

Simplify:

a $\dfrac{x^3}{x^{-3}}$ **b** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c** $(x^3)^{\frac{2}{3}}$ **d** $2x^{1.5} \div 4x^{-0.25}$ **e** $\sqrt[3]{125x^6}$ **f** $\dfrac{2x^2 - x}{x^5}$

Example 10

Evaluate:

a $9^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $25^{-\frac{3}{2}}$

Example 11

Given that $y = \frac{1}{16}x^2$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{2}}$ **b** $4y^{-1}$

1.5 Surds

If n is an integer that is **not** a square number, then any multiple of $\sqrt{n}$ is called a surd. Examples of surds are $\sqrt{2}$, $\sqrt{19}$ and $5\sqrt{2}$.

Surds are examples of **irrational numbers**. The decimal expansion of a surd is never-ending and never repeats, for example $\sqrt{2} = 1.414213562\ldots$

You can use surds to write exact answers to calculations.

- **You can manipulate surds using these rules:**
  - $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
  - $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

Example 12

Simplify:

a $\sqrt{12}$ b $\frac{\sqrt{20}}{2}$ c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

Example 13

Expand and simplify if possible:

a $\sqrt{2}(5 - \sqrt{3})$ b $(2 - \sqrt{3})(5 + \sqrt{3})$

1.6 Rationalising denominators

If a fraction has a surd in the denominator, it is sometimes useful to **rearrange** it so that the denominator is a **rational** number. This is called rationalising the denominator.

- **The rules to rationalise denominators are:**
  - **For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.**
  - **For fractions in the form $\frac{1}{a+\sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.**
  - **For fractions in the form $\frac{1}{a-\sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.**

Example 14

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$ b $\frac{1}{3+\sqrt{2}}$ c $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ d $\frac{1}{(1-\sqrt{3})^2}$

Mixed exercise 1

1 Simplify:

a $y^3 \times y^5$ b $3x^2 \times 2x^5$ c $(4x^2)^3 \div 2x^5$ d $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x + 3)(x - 5)$ b $(2x - 7)(3x + 1)$ c $(2x + 5)(3x - y + 2)$

3 Expand and simplify if possible:

a $x(x + 4)(x - 1)$ b $(x + 2)(x - 3)(x + 7)$ c $(2x + 3)(x - 2)(3x - 1)$

4 Expand the brackets:

a $3(5y + 4)$ b $5x^2(3 - 5x + 2x^2)$ c $5x(2x + 3) - 2x(1 - 3x)$ d $3x^2(1 + 3x) - 2x(3x - 2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$ **c** $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$ **c** $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

12 Expand and simplify if possible:

a $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$

13 Rationalise the denominator and simplify:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2} - 1}$ **c** $\frac{3}{\sqrt{3} - 2}$ **d** $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$ **e** $\frac{1}{(2 + \sqrt{3})^2}$ **f** $\frac{1}{(4 - \sqrt{7})^2}$

14 **a** Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b Hence, fully factorise $x^3 - x^2 - 17x - 15$.

E **15** Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

E/P **16** Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**

E **17** Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$. **(2 marks)**

E **18** Factorise completely $x - 64x^3$. **(3 marks)**

E/P **19** Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

E/P **20** Solve the equation $8 + x\sqrt{12} = \dfrac{8x}{\sqrt{3}}$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

P **21** A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

E **22** Show that $\dfrac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

E/P **23** Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

E/P **24** Given that $\dfrac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b. **(2 marks)**

Challenge

a Simplify $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$.

b Hence show that $\dfrac{1}{\sqrt{1} + \sqrt{2}} + \dfrac{1}{\sqrt{2} + \sqrt{3}} + \dfrac{1}{\sqrt{3} + \sqrt{4}} + \ldots + \dfrac{1}{\sqrt{24} + \sqrt{25}} = 4$

Self-Evaluation and further questions:

1. What topics from sections 1.1 to 1.6 (above) did you find difficult?

2. What topics in GCSE Mathematics do you normally do well at?

3. What topics in GCSE Mathematics do you enjoy?

4. What is your predicted Grade for GCSE Mathematics?

5. Why would you like to study Mathematics A Level?

6. Why have you chosen to study at Central St Michaels?

7. How would you describe your work ethic and are you committed to putting the time and hard work into your Mathematics A Level?

8. Please give details of any learning support that you require.

Thank you for completing all set tasks and I shall see you when the term begins. Please remember to bring in your completed work to your first day at Central St Michaels.

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